Derivation of a mobile impurity model for spin-charge separated Fermi systems

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Bad Honnef, June 2014

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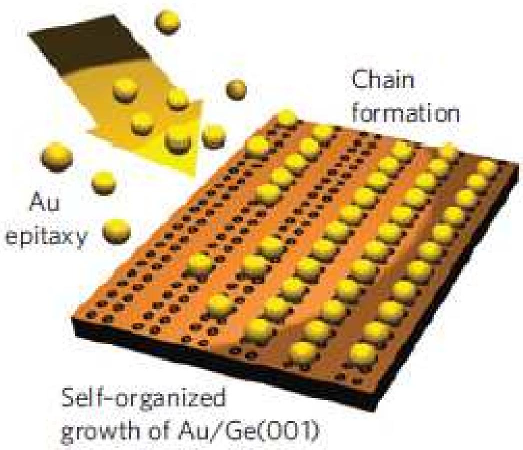
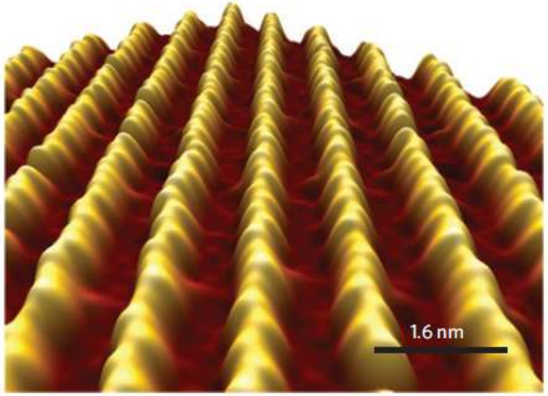
Interacting Electrons in 1D

- Self-organized atomic gold chains





Blumenstein et al, Nature Phys. 7 (2011)

- Large quantum fluctuations $\Rightarrow$ exotic behaviour

  Landau's principle of adiabatic continuity breaks down

  Interactions dominate physical properties

1D Metals: low energy limit

- Universality class: **Luttinger liquid**
- Quantum numbers are not robust

  Elementary excitations bosonic
- Characteristic power-law behaviour

  Spin-charge separation $\Rightarrow$ signatures in tunneling experiments

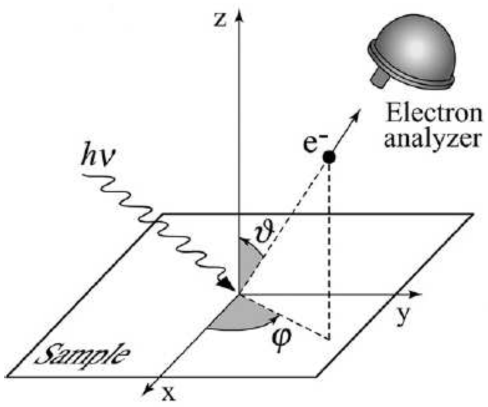
Angle-resolved photoemission

- Single-particle spectral function:

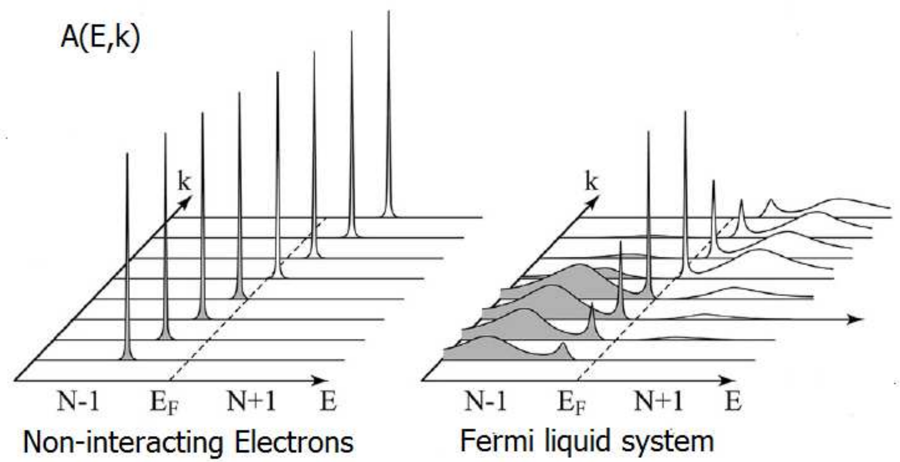
$$A(\omega, k) = -\frac{1}{\pi}\text{Im}\int_0^\infty e^{i\omega t} G(k,t)\, dt$$

$$G(k,t) = -i\int dx\, e^{-ikx}\langle\{\psi_\uparrow^\dagger(x,t), \psi_\uparrow(0,t)\}\rangle$$

- Geometry



- Spectra in 3D

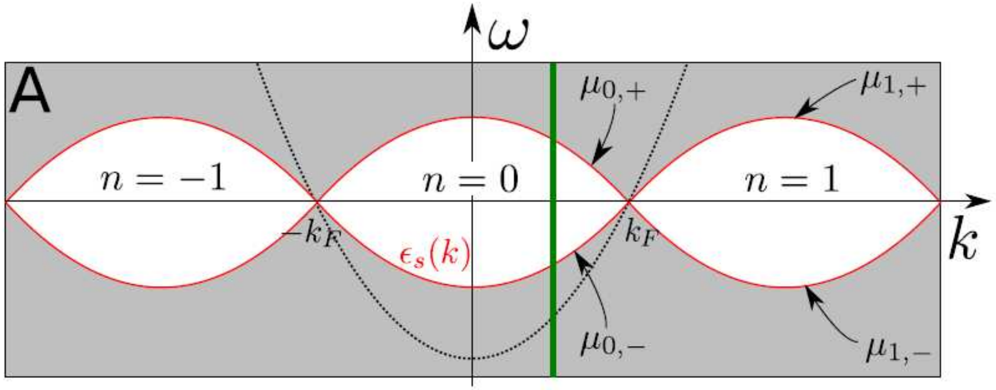


- 1D: 2 dispersing singular features, power-law behaviour

Single-particle spectral function in 1D

$$A(\omega, k) = \sum_m |\langle \omega_m | \psi_\uparrow^\dagger(k) | 0 \rangle|^2 \, \delta(\omega - \omega_m) \qquad \text{for} \quad \omega > 0$$

- Support $A(\omega, k)$ for commensurate fillings

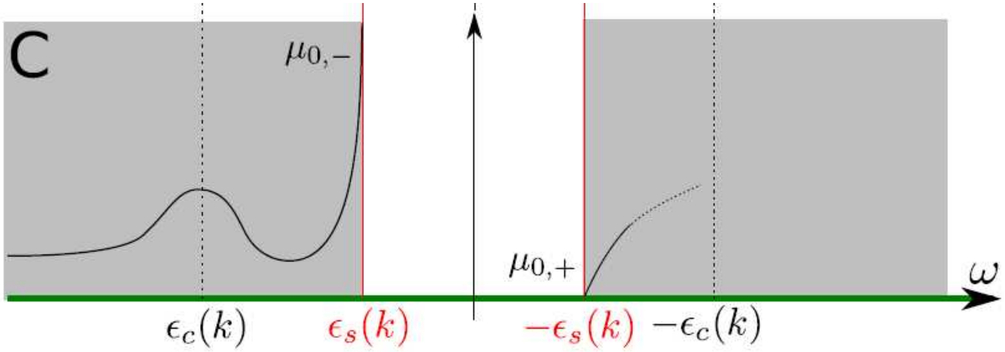


$\epsilon_s(k)$: spinon dispersion

$\epsilon_c(k)$: holon dispersion

Figs.: *T. L. Schmidt et al, PRL (2010)*

- Power-laws at thresholds



Interest in the time-dependent Green's function

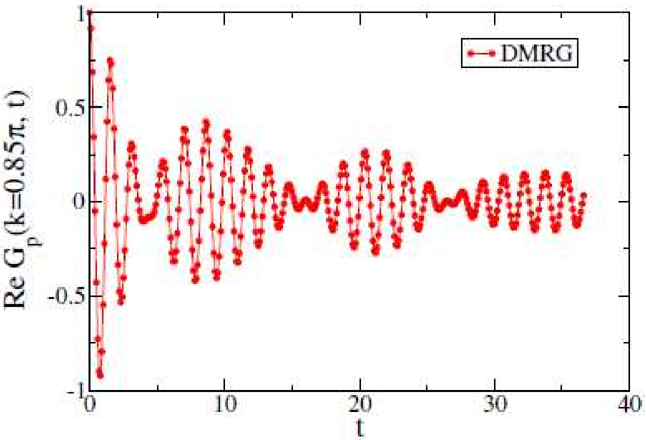
- Time-dependent DMRG methods

  G. Vidal, PRL 91 (2003); ibid. 93 (2004)

  A. J. Daley et al, J. Stat. Mech. (2004)

  S. R. White and A. E. Feiguin, PRL 93 (2004)

  Fig.: *R.G. Pereira, Phys. Rev. B (2009)*



- Ultracold atoms on optical lattices

  1D Fermi gas: **H. Moritz et al, PRL 94 (2005)**

  ARPES like: **J. T. Stewart et al, Nature 454 (2008); M. Feld et al, Nature 480 (2011)**

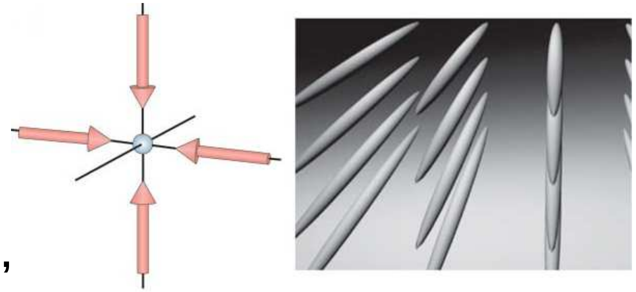


Fig.: *I. Bloch, Nature Phys. (2005)*

Non-equilibrium-dynamics: **M. Greiner et al, Nature 419 (2002); T. Kinoshita et al, Nature 440 (2006); S. Hofferberth et al, Nature 449 (2007); S. Trotzky et al, Nature Phys. (2012)**

Luttinger liquid

- Two free bosons

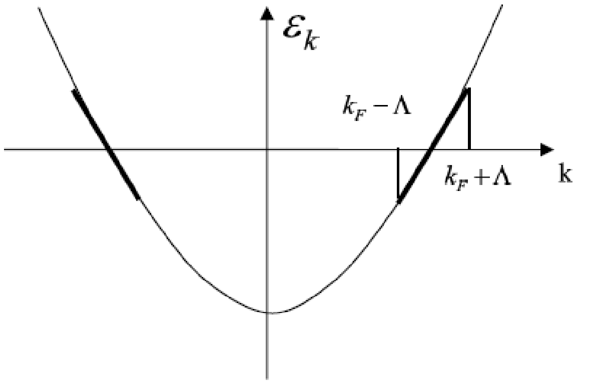
Velocities v_s, v_c, Luttinger parameters K_s, K_c

$$H = H_c + H_s \qquad H_\nu = \frac{v_\nu}{2}\int dx \left[K_\nu(\partial_x\Theta_\nu)^2 + \frac{1}{K_\nu}(\partial_x\phi_\nu)^2\right]$$

- Fermion operator:

$$\psi_\uparrow(x) \sim e^{ik_F x}\psi_{R,\uparrow}(x) + e^{-ik_F x}\psi_{L,\uparrow}(x)$$

$$\psi_{R,\uparrow}(x) = \frac{1}{\sqrt{2\pi a}} e^{i\sqrt{2\pi}\phi_{R,c}(x)} e^{i\sqrt{2\pi}\phi_{R,s}(x)}$$



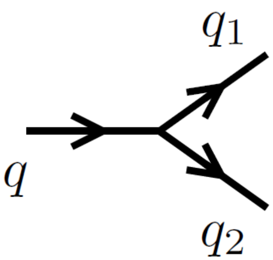
Higher order band curvature terms neglected

- Spectral function: **V. Meden and K. Schönhammer, Phys. Rev. B 46 (1992); J. Voit, ibid. 47 (1993)**

Band curvature terms

- Band curvature spinless for simplicity

irrelevant operator



Interaction vertex

$$\delta H = \int dx \Big\{ \eta_- [(\partial_x \phi_L)^3 - (\partial_x \phi_R)^3] + \eta_+ [(\partial_x \phi_L)^2 \partial_x \phi_R - (\partial_x \phi_R)^2 \partial_x \phi_L] \Big\}$$

- First-order correction to the single-particle Green's function

$$\frac{\delta G_R(x,t)}{G_R(x,t)} = \frac{\lambda^3 \eta_-}{iv} \left[\frac{1}{x - vt} - \frac{x + vt}{(x - vt)^2} \right] + \frac{\bar{\lambda}^3 \eta_-}{iv} \left[\frac{1}{x + vt} - \frac{x - vt}{(x + vt)^2} \right]$$

H. Karimi and I. Affleck, PRB 84 (2011)

Singular contributions at the light cone $x = \pm vt$

Nonlinear Luttinger liquids (spinless)

- Couple Luttinger liquid to mobile impurity

$$\psi(x) \sim \underbrace{e^{ik_F x}\psi_R(x) + e^{-ik_F x}\psi_L(x)}_{\text{low-energy modes near } \pm k_F} + \underbrace{e^{ikx} d}_{\text{high-energy particle}}$$

M. Khodas et al., PRB 76 (2007), A. Imambekov and L.I. Glazman, Science 323 (2009);

A. Imambekov, T.L. Schmidt, and L.I. Glazman, Rev. Mod. Phys. 84 (2012)

- Weak coupling expansion **R. G. Pereira, S. R. White, and I. Affleck, PRB 79 (2009)**

$$\begin{aligned} H &= \frac{v}{2}\int dx \left[K(\partial_x\Theta)^2 + \frac{1}{K}(\partial_x\phi)^2\right] + d^\dagger(\epsilon - iu\partial_x)d \\ &\quad + \left(\frac{\kappa_L - \kappa_R}{2\sqrt{\pi}}\partial_x\Theta + \frac{\kappa_L + \kappa_R}{2\sqrt{\pi}}\partial_x\Phi\right) d^\dagger d \end{aligned}$$

- $G(k,t) \sim \int dx \langle d(x,t) d^\dagger(0,0)\rangle \sim e^{-i\epsilon t}\left[\frac{i}{(u-v)t+i\eta}\right]^{\nu_R}\left[\frac{-i}{(u+v)t-i\eta}\right]^{\nu_L}$

Mobile impurity model for spinful Fermions

- Phenomenological: **T. L. Schmidt, A. Imambekov, and L. I. Glazman, PRL 104 (2010)**
- Constructive derivation using a weak coupling expansion

Bosonization: $\psi_{R,\uparrow}(x) = e^{i\sqrt{2\pi}\phi_{R,c}(x)} e^{i\sqrt{2\pi}\phi_{R,s}(x)}$

Refermionization: $\psi_{R,\uparrow}(x) \sim \underbrace{\psi_{R,c}(x)}_{\text{carries charge of free Fermion}} \underbrace{e^{i \text{ string operator}}}_{\text{carries no charge}} \psi_{R,s}(x) e^{i \text{ string operator}}$

where $\psi_{R,c}(x) = e^{i\sqrt{4\pi}\tilde{\phi}_{R,c}(x)}$ and $e^{i \text{ string operator}} = e^{i\lambda\sqrt{\pi}\tilde{\phi}_c(x)}$

Project $\psi_{R,s}(x) \sim \underbrace{\tilde{\psi}_{R,s}(x)}_{\text{low energy modes}} + e^{-ikx} \underbrace{d}_{\text{high energy}}$

- New Fermions $\psi_{R,c/s}(x)$ generally interacting

Luther-Emery point

- Non-interacting Dirac Fermions for $K_c = K_s = \frac{1}{2}$

A. Luther and V.J. Emery, Phys. Rev. Lett. 33, 589 (1974)

$$H = \sum_{\nu=c,s} \int dx \left[\psi^\dagger_{R,\nu}(x)(-iv_\nu \partial_x)\psi_{R,\nu}(x) + \psi^\dagger_{L,\nu}(x)(iv_\nu \partial_x)\psi_{L,\nu}(x) \right]$$

- Good starting point for weak-coupling expansion

- All perturbing operators up to scaling dimension 4 identified

1) Spinon mass: $g_1 \ \left[\psi^\dagger_{R,s}\psi_{L,s} + \text{H.c.} \right] \propto g_1 \ \cos(\sqrt{4\pi}\phi_s)$

2) Marginal: $g_2 \, n_c(x) \left[\psi^\dagger_{R,s}\psi_{L,s} + \text{H.c.} \right] \propto g_2 \, \partial_x \phi_c \cos(\sqrt{4\pi}\phi_s)$

Lattice realization of Luther-Emery point

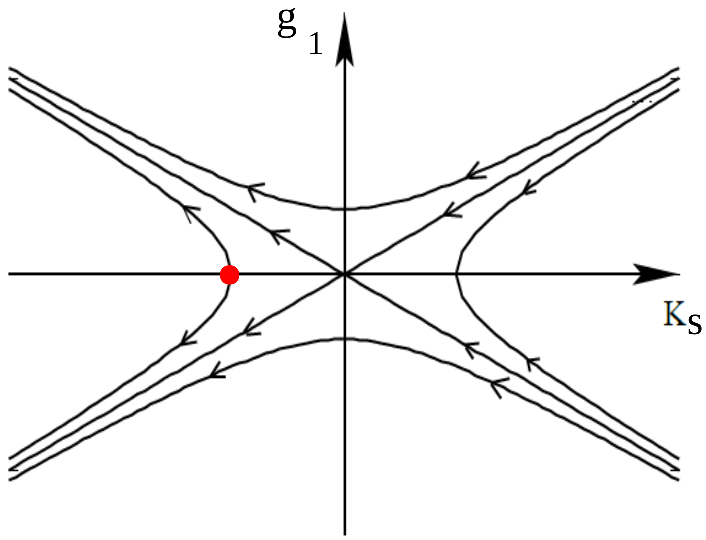
- Extended Hubbard model with spin-anisotropic interaction

$$
\begin{aligned}
H \;=\; & -\sum_{j=1}^{L-1}\sum_{\sigma}\left[c^{\dagger}_{j,\sigma}c_{j+1,\sigma} + \text{H.c.}\right] + U\sum_{j=1}^{L} n_{j,\uparrow}n_{j,\downarrow} + V_1\sum_{j=1}^{L-1} n_j n_{j+1} \\
& + V_2\sum_{j=1}^{L-2} n_j n_{j+2} + J_z\sum_{j=1}^{L-1} S^z_j S^z_{j+1}
\end{aligned}
$$

- Tune to Luttinger liquid with $K_c = K_s = \frac{1}{2}$

  Tune spinon mass g_1 to zero

  v_s, v_c, K_s, K_c determined by excitation spectrum



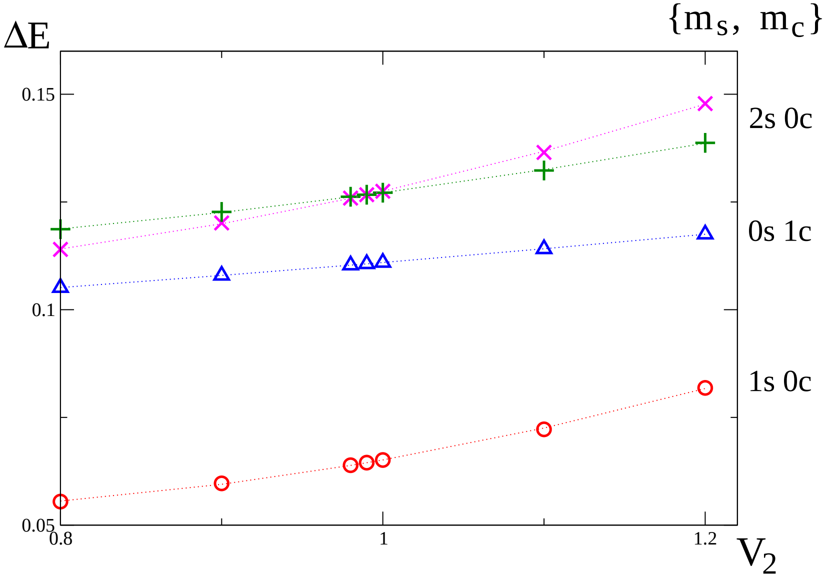
Fine-tuning of the spinon mass g_1

$$H = -\sum_{j=1}^{L-1}\sum_{\sigma}\left[c^{\dagger}_{j,\sigma}c_{j+1,\sigma} + \text{H.c.}\right] + U\sum_{j=1}^{L} n_{j,\uparrow}n_{j,\downarrow} + V_1\sum_{j=1}^{L-1} n_j n_{j+1}$$
$$+V_2\sum_{j=1}^{L-2} n_j n_{j+2} + J_z\sum_{j=1}^{L-1} S^z_j S^z_{j+1}$$

- $V_1 = J_z = 0$

  Level crossing tuned by V_2

  U=3, L=64, n=1/4



- Fine-tune V_1 and J_z to reach $K_s = K_c = \frac{1}{2}$

RG-analysis of marginal operator

- Expand around Luther-Emery point $K_\nu = \frac{1}{2} - \frac{g_\nu}{\upsilon_\nu}$

$$S = S_0 + g_2 \int_x \mathcal{O}_2(x,\tau) + g_c \int_x \mathcal{O}_c(x,\tau) + g_s \int_x \mathcal{O}_s(x,\tau)$$

$$\begin{aligned}
\mathcal{O}_2(x,\tau) &= \partial_x \phi_c(x,\tau) \cos(\sqrt{4\pi}\phi_s(x,\tau)) \\
\mathcal{O}_c(x,\tau) &= 2\partial_x \phi_{R,c}(x,\tau)\partial_x \phi_{L,c}(x,\tau) \\
\mathcal{O}_s(x,\tau) &= 2\partial_x \phi_{R,s}(x,\tau)\partial_x \phi_{L,s}(x,\tau)
\end{aligned}$$

$\mathcal{O}_2(x,\tau)$ has non-vanishing conformal spin

- Beta-functions: $\beta(g_i) = a\frac{\partial}{\partial a} g_i \left(\{g_j^0\}, a\right)$

$$\begin{aligned}
\beta(g_2) &= -\frac{1}{\upsilon_s} g_2 g_s \\
\beta(g_c) &= \frac{\pi}{2\upsilon_s} (g_2)^2 \\
\beta(g_s) &= 0
\end{aligned}$$

Including 2-loop RG:
g_s not flowing

RG equations for marginal operator

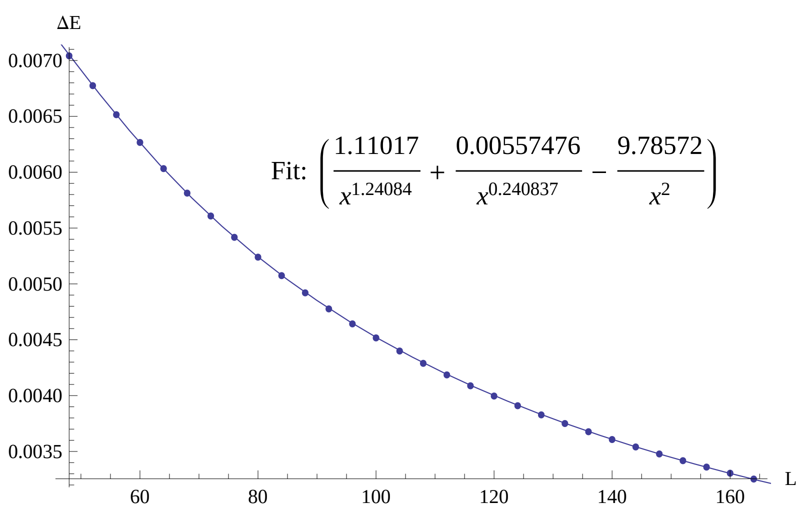
- Solve RG-equations:

$$g_2(\Lambda) = g_2(\Lambda_0)\left(\frac{\Lambda}{\Lambda_0}\right)^{2\delta K_s}$$

low-energy limit $\ln \Lambda/\Lambda_0 \to \infty$

$$g_c(\Lambda) = g_c(\Lambda_0) + \frac{[g_2(\Lambda_0)]^2}{8g_s}\left[1 - \left(\frac{\Lambda}{\Lambda_0}\right)^{4\delta K_s}\right]$$

- Fit function: $f(L) = a_0\left(\frac{1}{L}\right)^{2K_s-1} + a_1\frac{1}{L} + a_2\left(\frac{1}{L}\right)^{2K_s} + a_3\left(\frac{1}{L}\right)^2$



$$\Delta E_0(S_z = 1, 0) - \tfrac{1}{4}\Delta E_0(S_z = 2, 0)$$

$U = 3$, $V_1 = 0.85$, $V_2 = 1.1$, $J_z = 5.2$

Conclusion

- Mobile-impurity model for spin-charge separated Fermi systems derived

  Applications for spectral function, structure factor, optical conductivity

- Weak-coupling expansion starting from Luther-Emery point

- Lattice realization of Luther-Emery point numerically (identified)

  Marginal operator present $\rightarrow$ include in calculation of correlators

Energy spectrum for open boundaries

- Descendants: $E_{m_s,m_c}(S^z,\Delta N) - E_0(S^z,\Delta N) = (m_s \upsilon_s + m_c \upsilon_c)\frac{\pi}{L}$

  Each spin/charge mode $\{m_s, m_c\}$ has several states with nearly the same energy

- Charge excitations $\Delta N = \pm 2$ $\boxed{\frac{\pi}{2}\frac{\upsilon_c}{K_c} = \frac{1}{n^2\kappa}}$ filling $n \equiv \frac{N}{L} = \frac{1}{4}$

$$\frac{1}{\kappa} = \frac{n^2 L}{4}\Big(E_0(0,\Delta N = 2) + E_0(0,\Delta N = -2) - 2E_0\Big)$$

- Spin excitations $S_z = \pm 1$ $\boxed{\frac{\pi}{2}\frac{\upsilon_s}{K_s} = \frac{\Delta_s}{2}L}$

$$\Delta_s = E_0(S_z = 1, 0) - E_0$$