Temperature driven crossover in the 1D Bose gas

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Outline

1. The Lieb-Liniger model
2. Correlation lengths via the QTM method
3. The LL model as the continuum limit of the XXZ spin chain
4. The field-field correlator. Correlation lengths
5. The density-density correlator. Correlation lengths and crossover

The Lieb-Liniger Model

The Hamiltonian (Lieb-Liniger 1963)

$$H_{NLS} = \int_0^l dx \left[\partial_x \Psi^\dagger(x)\partial_x\Psi(x) + c\Psi^\dagger(x)\Psi^\dagger(x)\Psi(x)\Psi(x) - \mu\Psi^\dagger(x)\Psi(x)\right] ,$$

$c > 0$ coupling constant, μ chemical potential, l the length of the system ($\hbar = 2m = 1$) with m the mass of the particles.

$$[\Psi(x), \Psi^\dagger(x')] = \delta(x - x') , \quad [\Psi(x), \Psi(x')] = [\Psi^\dagger(x), \Psi^\dagger(x')] = 0$$

$$H_{NLS} = -\sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2} + c \sum_{1\leq i<j\leq n} \delta(x_i - x_j) - \mu n$$

Bethe Anstaz equations

$$e^{ik_j l} = \prod_{s\neq j}^{n} \frac{k_j - k_s + ic}{k_j - k_s - ic} , \quad j = 1, \cdots , n$$

Energy spectrum $E(\{k\}) = \sum_{j=1}^{n} e_0(k_j) , \quad e_0(k) = k^2 - \mu ,$

Thermodynamics and correlation functions

Grand-canonical potential per unit length (Yang and Yang 1969):

$$\phi(\mu, T) = -\frac{T}{2\pi}\int_{-\infty}^{+\infty} \log\left(1 + e^{-\varepsilon(k)/T}\right) dk$$

Yang-Yang equation:

$$\varepsilon(k) = k^2 - \mu - \frac{T}{2\pi}\int_{\mathbb{R}} K(k-k')\log\left(1 + e^{-\varepsilon(k')/T}\right) dk' \,.$$

$\theta(k) = i\log\left(\frac{ic+k}{ic-k}\right)$, $\quad \lim_{k\to\pm\infty}\theta(k) = \pm\pi$; $\quad K(k-k') = \frac{d}{dk}\theta(k-k') = \frac{2c}{(k-k')^2+c^2}$

Temperature dependent correlation functions

$$\langle\mathcal{O}\rangle_T = \frac{\sum\langle\Omega|\mathcal{O}|\Omega\rangle e^{-E/T}}{\sum e^{-E/T}} \,,$$

- Field-field correlation function: $\langle\Psi^\dagger(x)\Psi(0)\rangle_T$
- Density-density correlation function: $\langle j(x)j(0)\rangle_T$ with $j(x) = \Psi^\dagger(x)\Psi(x)$

Previous results

Impenetrable limit ($c \to \infty$)

- Girardeau (1960); Lenard (1964),(1966); Vaidya and Tracy (1979); Jimbo, Miwa, Môri, Sato (1980); Its, Izergin, Korepin, Slavnov,Varzugin (1989-1993); Gangardt (2004)

Finite coupling strength

- TLL/CFT: Haldane (1981); Bogoliubov, Izergin and Korepin (1986); Berkovich and Murthy (1988)
- ABA: Bogoliubov and Korepin (1984); Izergin and Korepin (1984); Kitanine, Kozlowski, Maillet, Slavnov and Terras (2009),(2012); Kozlowski, Maillet and Slavnov (2011), Kozlowski and Terras (2011), Kozlowski (2011)
- Numerical-ABA: Caux, Calabrese (2006); Caux, Calabrese, Slavnov (2007); Panfil, Caux (2014)

Method

Integrable lattice models at $T > 0$: Quantum Transfer Matrix

- Largest eigenvalue of QTM $\rightarrow$ Free energy of the system $F = -k_B T \log \Lambda_0(0)$
- Next largest eigenvalues $\rightarrow$ Correlation lengths

Problem: The QTM does not exist for continuum models!

- Continuum limit of the XXZ spin chain $\rightarrow$ Lieb-Liniger model (Kulish, Sklyanin 1979)
- Yang's thermodynamics from the XXZ spin chain QTM result (Seel, Bhattacharyya, Göhmann, Klümper 2007)
- Multiple integral representation for the correlation functions (Seel, Bhattacharyya, Göhmann, Klümper 2007; Seel, Göhmann, Klümper 2008)

General strategy: Obtain asymptotic expansions for the correlation functions of the XXZ spin chain and take the continuum limit.

The XXZ spin chain

The Hamiltonian: $H(J,\Delta,h) = H^{(0)}(J,\Delta) - hS_z$

$$H^{(0)}(J,\Delta) = J\sum_{j=1}^{L}\left[\sigma_x^{(j)}\sigma_x^{(j+1)} + \sigma_y^{(j)}\sigma_y^{(j+1)} + \Delta(\sigma_z^{(j)}\sigma_z^{(j+1)} - 1)\right], \quad S_z = \frac{1}{2}\sum_{j=1}^{L}\sigma_z^{(j)}$$

$\Delta = \cos\eta$ with $0 < \eta < \pi (|\Delta| < 1)$; $h < h_c = 8J\cos^2(\eta/2)$

Bethe Ansatz equations: $\left(\frac{\sinh(\lambda_j - i\eta/2)}{\sinh(\lambda_j + i\eta/2)}\right)^L = \prod_{s\neq j}^{n} \frac{\sinh(\lambda_j-\lambda_s-i\eta)}{\sinh(\lambda_j-\lambda_s+i\eta)}, \qquad j = 1, \cdots, n$

Energy spectrum: $\overline{E}(\{\lambda\}) = \sum_{j=1}^{n}\overline{e}_0(\lambda_j) - h\frac{L}{2}, \quad \overline{e}_0(\lambda) = \frac{2J\sinh^2(i\eta)}{\sinh(\lambda+i\eta/2)\sinh(\lambda-i\eta/2)} + h$

Bare momentum: $p_0(\lambda) = i\log\left(\frac{\sinh(i\eta/2+\lambda)}{\sinh(i\eta/2-\lambda)}\right)$,

Scattering phase and kernel:

$\overline{\theta}(\lambda) = i\log\left(\frac{\sinh(i\eta+\lambda)}{\sinh(i\eta-\lambda)}\right), \overline{K}(\lambda) = \overline{\theta}'(\lambda) = \frac{\sin(2\eta)}{\sinh(\lambda+i\eta)\sinh(\lambda-i\eta)}$

Continuum limit

XXZ spin chain	One-dimensional Bose gas
lattice constant $\delta = \mathcal{O}(\epsilon^2)$	
number of lattice sites $L = \mathcal{O}(1/\epsilon^2)$	physical length $l = L\delta$
interaction strength $J = 1/2$	particle mass $m = 1/2$
magnetic field $h = \mathcal{O}(\epsilon^2)$	chemical potential $\mu = (\frac{\epsilon^2}{\delta^2} + \frac{\epsilon^4}{4\delta^2} - \frac{h}{\delta^2})$
anisotropy $\Delta = \cos\eta = \epsilon^2/2 - 1$	repulsion strength $c = \epsilon^2/\delta$
inverse temperature $\overline{\beta}$	inverse temperature $\beta = \overline{\beta}\delta^2$

Seel, Bhattacharyya, Göhmann, Klümper (2007); O.I.P. and Klümper (2013)
Spectral parameter $\frac{\epsilon}{\delta}\lambda = k$; $\eta = \pi - \epsilon$

- BAE (XXZ spin chain) $\rightarrow$ BAE (Bose gas)
- One-particle momentum and two-particle scattering
  $p_0(\lambda) \rightarrow \delta k\,, \quad \overline{\theta}(\lambda) \rightarrow -\theta(k)\,, \quad \overline{K}(\lambda) \rightarrow -\frac{\epsilon}{\delta}K(k)\,,$
- One-particle energy $\overline{\beta}\overline{e}_0(\lambda) \rightarrow \beta e_0(k)$
- $Z_{XXZ}(h,\beta) \equiv \lim_{L\rightarrow\infty}\sum_{\{\lambda\}} e^{-\overline{\beta}\overline{E}(\{\lambda\})} \rightarrow Z_{NLS}(\mu,\beta) \equiv \lim_{l\rightarrow\infty}\sum_{\{k\}} e^{-\beta E(\{k\})}$.

XXZ spin chain correlators at low-T and vanishing magnetic field $\rightarrow$ temperature dependent correlators in the Bose gas at all T

The XXZ spin chain QTM

XXZ spin chain R-matrix:

$$R(\lambda,\mu)=\begin{pmatrix}1&0&0&0\\0&b(\lambda,\mu)&c(\lambda,\mu)&0\\0&c(\lambda,\mu)&b(\lambda,\mu)&0\\0&0&0&1\end{pmatrix}\quad \begin{matrix}b(\lambda,\mu)=\frac{\sinh(\lambda-\mu)}{\sinh(\lambda-\mu+i\eta)}\\ c(\lambda,\mu)=\frac{\sinh(i\eta)}{\sinh(\lambda-\mu+i\eta)}\end{matrix}$$

L- operators:

$$L_j(\lambda,-u')=\sum_{a,b,a_1,b_1=1}^{2}R_{b\,b_1}^{aa_1}(\lambda,-u')e_{ab}^{(0)}e_{a_1b_1}^{(j)}\,,\quad \tilde{L}_j(u',\lambda)=\sum_{a,b,a_1,b_1=1}^{2}R_{a_1\,b}^{b_1\,a}(u',\lambda)e_{ab}^{(0)}e_{a_1b_1}^{(j)}\,,$$

$u'=-2iJ\sin\eta\frac{\beta}{N}$; N is the Trotter number;
$e_{ab}^{(0)}=e_{ab}\otimes\mathbb{I}_2^{\otimes L}$ and $e_{ab}^{(i)}=\mathbb{I}_2\otimes\mathbb{I}_2^{\otimes(i-1)}\otimes e_{ab}\otimes\mathbb{I}_2^{\otimes(N-i)}$
Monodromy matrix:

$$T^{QTM}(\lambda)=L_N(\lambda,-u')\tilde{L}_{N-1}(u',\lambda)\cdots L_2(\lambda,-u')\tilde{L}_1(u',\lambda)$$

The QTM:

$$t^{QTM}(\lambda)=\mathrm{tr}_0T^{QTM}(\lambda)$$

The QTM spectrum and correlation functions

The eigenvalues of the QTM:

$$\Lambda(\lambda) = b(u',\lambda)^{N/2} e^{\beta h/2} \prod_{j=1}^{p} \frac{\sinh(\lambda-\lambda_j-i\eta)}{\sinh(\lambda-\lambda_j)} + b(\lambda,-u')^{N/2} e^{-\beta h/2} \prod_{j=1}^{p} \frac{\sinh(\lambda-\lambda_j+i\eta)}{\sinh(\lambda-\lambda_j)}$$

Bethe ansatz equations: $\left(\frac{b(u',\lambda_j)}{b(\lambda_j,-u')}\right)^{N/2} = e^{-\beta h} \prod_{j\neq k}^{p} \frac{\sinh(\lambda_j-\lambda_k+i\eta)}{\sinh(\lambda_j-\lambda_k-i\eta)}\,, \quad j=1,\cdots,p\,.$

Largest eigenvalue in the $N/2$ sector ($p=N/2$): $F=-k_B T\log\Lambda_0(0)$

Longitudinal correlation ($m\to\infty$)

$$\langle\sigma_z^{(1)}\sigma_z^{(m+1)}\rangle_T = const + \sum_{i\in N/2 \text{ sector}} A_i\, e^{-\frac{m}{\xi_i^{(d)}}}\,, \quad 1/\xi_i^{(d)} = \log(\Lambda_0(0)/\Lambda_i^{(ph)}(0))$$

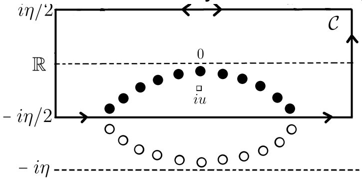
Transversal correlation ($m\to\infty$)

$$\langle\sigma_+^{(1)}\sigma_-^{(m+1)}\rangle_T = \sum_{i\in N/2-1 \text{ sector}} B_i\, e^{-\frac{m}{\xi_i^{(s)}}}\,, \quad 1/\xi_i^{(s)} = \log(\Lambda_0(0)/\Lambda_i^{(s)}(0))$$

Continuum limit $\langle\sigma_z^{(1)}\sigma_z^{(m+1)}\rangle_T \to \langle j(x)j(0)\rangle_T$; $\langle\sigma_+^{(1)}\sigma_-^{(m+1)}\rangle_T \to \langle\Psi^\dagger(x)\Psi(0)\rangle_T$

Bose gas thermodynamics from the XXZ QTM. I

NLIE for the auxiliary function, Integral expresion for the eigenvalue



$$\log \mathfrak{a}(\lambda) = -\beta h - \beta \frac{2J \sinh^2(i\eta)}{\sinh(\lambda + i\eta)\sinh\lambda} - \frac{1}{2\pi}\int_{\mathcal{C}} \overline{K}(\lambda-\mu)\log(1+\mathfrak{a}(\mu))d\mu$$

$$\log \Lambda_0(0) = \frac{\beta h}{2} + \frac{1}{2\pi}\int_{\mathcal{C}} \frac{\sin\eta}{\sinh(\mu+i\eta)\sinh(\mu)}\log(1+\mathfrak{a}(\mu))d\mu\,.$$

At low-T neglect the upper part of the contour; $e^{-\overline{\varepsilon}(\lambda)/T} = \mathfrak{a}(\lambda - i\eta/2)$, Klümper and Scheeren (2003); Dugave, Göhmann, Kozlowski (2013)

$$\overline{\varepsilon}(\lambda) = \overline{e}_0(\lambda) + \frac{T}{2\pi}\int_{\mathbb{R}} \overline{K}(\lambda-\mu)\log\left(1+e^{-\overline{\varepsilon}(\mu)/T}\right)d\mu\,,$$

$$\log \Lambda_0(0) = \frac{h}{2T} + \frac{1}{2\pi}\int_{\mathbb{R}} p_0'(\lambda)\log\left(1+e^{-\overline{\varepsilon}(\lambda)/T}\right)d\lambda$$

Bose gas thermodynamics from the XXZ QTM. II

XXZ spin chain (low T)

$$\log \Lambda_0(0) = \frac{h}{2\overline{T}} + \frac{1}{2\pi} \int_{\mathbb{R}} p_0'(\lambda) \log\left(1 + e^{-\overline{\varepsilon}(\lambda)/\overline{T}}\right) d\lambda$$

$$\overline{\varepsilon}(\lambda) = \overline{e}_0(\lambda) + \frac{\overline{T}}{2\pi} \int_{\mathbb{R}} \overline{K}(\lambda - \mu) \log\left(1 + e^{-\overline{\varepsilon}(\mu)/\overline{T}}\right) d\mu$$

Using $\phi(\mu, T) = (f(h, \overline{T}) + h/2)/\delta^3$ and $f(h, \overline{T}) = -\overline{T} \log \Lambda_0(0)$

Bose gas (all T)

$$\phi(\mu, T) = -\frac{T}{2\pi} \int_{-\infty}^{+\infty} \log\left(1 + e^{-\varepsilon(k)/T}\right) dk$$

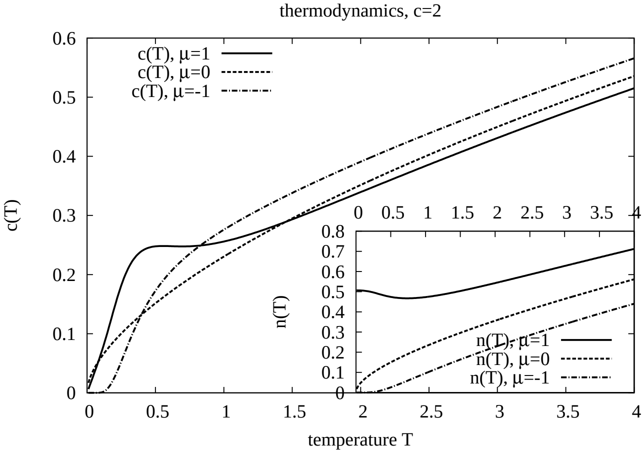
$$\varepsilon(k) = k^2 - \mu - \frac{T}{2\pi} \int_{\mathbb{R}} K(k - k') \log\left(1 + e^{-\varepsilon(k')/T}\right) dk' \,.$$

Seel, Bhattacharyya, Göhmann, Klümper (2007)

Bose gas thermodynamics. Numerical results

Grand-canonical potential per unit length: $\phi(\mu, T) = -\frac{T}{2\pi}\int_{-\infty}^{+\infty} \log\left(1 + e^{-\varepsilon(k)/T}\right) dk$

Yang-Yang equation: $\varepsilon(k) = k^2 - \mu - \frac{T}{2\pi}\int_{\mathbb{R}} K(k-k') \log\left(1 + e^{-\varepsilon(k')/T}\right) dk'$.



Specific heat and particle density

- $\mu < 0$: gas phase $c(T), n(T) \simeq e^{-\frac{|\mu|}{T}}$
- $\mu = 0$: critical value $c(T), n(T) \simeq T^{1/2}$
- $\mu > 0$: CFT $c(T) \simeq T$, $n(T) \simeq const.$

Asymptotics of the field-field correlator

Asymptotic expansion (Klümper and OIP 2013)

$$\langle \Psi^\dagger(x)\Psi(0)\rangle_T = \sum_i B_i\, e^{-\frac{x}{\xi^{(s)}[v_i]}}\,, \qquad x \to \infty$$

Correlation lengths

$$\frac{1}{\xi^{(s)}[v_i]} = -\frac{1}{2\pi}\int_{\mathbb{R}} \log\left(\frac{1+e^{-v_i(k)/T}}{1+\,e^{-\varepsilon(k)/T}}\right) dk - ik_0 - i\sum_{j=1}^{r} k_j^+ + i\sum_{j=1}^{r} k_j^-$$

$$v_i(k) = k^2 - \mu \pm i\pi T + iT\theta(k-k_0) + iT\sum_{j=1}^{r}\theta(k-k_j^+) - iT\sum_{j=1}^{r}\theta(k-k_j^-)$$
$$-\frac{T}{2\pi}\int_{\mathbb{R}} K(k-k')\log\left(1+e^{-v_i(k')/T}\right) dk'$$

$2r+1$ parameters: k_0 and $\{k_j^+\}_{j=1}^r$ ($\{k_j^-\}_{j=1}^r$) upper (lower) half of the complex plane; plus (minus) sign when k_0 is in the first (second) quadrant

$$1+e^{-v_i(k_0)/T} = 0\,, \quad 1+e^{-v_i(k_j^{\pm})/T} = 0.$$

At low temperatures: k_0 can dive under the real axis; indented contour such that k_0 is above it

Field-field correlator $\langle\Psi^\dagger(x)\Psi(0)\rangle_T$: Leading term I

Correlation length

$$\frac{1}{\xi^{(s)}[v]}=-\frac{1}{2\pi}\int_{\mathbb{R}}\log\left(\frac{1+e^{-v(k)/T}}{1+e^{-\varepsilon(k)/T}}\right)dk-ik_0$$

NLIE

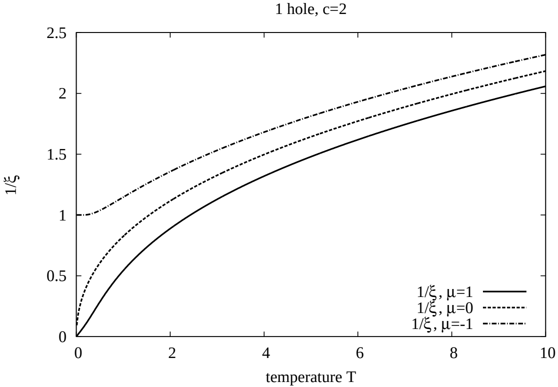
$$v(k)=k^2-\mu+i\pi T+iT\theta(k-k_0)-\frac{T}{2\pi}\int_{\mathbb{R}}K(k-k')\log\left(1+e^{-v(k')/T}\right)dk'$$

- $\mu>0$: k_0 is located in the upper half-plane at intermediate and high temperatures, at low-temperatures located in the lower half-plane
- $\mu<0$: k_0 is imaginary

CFT ($\mu>0$)

$$\langle\Psi^\dagger(x)\Psi(0)\rangle_T=B_0\left(\frac{\pi T/v_F}{\sinh(\pi Tx/v_F)}\right)^{\frac{1}{2\mathcal{Z}^2}}+\sum_{l=\pm1,\cdots}B_l\,e^{2ixlk_F}\left(\frac{\pi T/v_F}{\sinh(\pi Tx/v_F)}\right)^{\frac{1}{2\mathcal{Z}^2}+2l^2\mathcal{Z}^2}$$

Field-field correlator $\langle\Psi^\dagger(x)\Psi(0)\rangle_T$: Leading term II



- $\mu < 0$: gas phase
  $1/\xi(T)$ finite
- $\mu = 0$: critical value
  $1/\xi(T) \simeq T^{1/2}$
- $\mu > 0$: CFT
  $1/\xi(T) \simeq \frac{\pi T}{v_F} \frac{1}{2\mathcal{Z}^2}$

Impenetrable limit ($c \to \infty$): Its, Izergin and Korepin (1993)

$$\frac{1}{\xi} = \frac{1}{2\pi} \int_{\mathbb{R}} \log\left(\frac{e^{(k^2-\mu)/T}+1}{e^{(k^2-\mu)/T}-1}\right) dk + \sqrt{|\mu|}\,, \qquad \mu < 0$$

$$\frac{1}{\xi} = \frac{1}{2\pi} \int_{\mathbb{R}} \log\left|\frac{e^{(k^2-\mu)/T}+1}{e^{(k^2-\mu)/T}-1}\right| dk\,, \qquad \mu > 0$$

Asymptotics of the density-density correlator

Asymptotic expansion (Kozlowski, Maillet and Slavnov 2010)

$$\langle j(x)j(0)\rangle_T = const + \sum_i A_i\, e^{-\frac{x}{\xi^{(d)}[u_i]}}\,, \qquad x\to\infty\,,$$

Correlation lengths

$$\frac{1}{\xi^{(d)}[u_i]} = -\frac{1}{2\pi}\int_{\mathbb{R}} \log\left(\frac{1+e^{-u_i(k)/T}}{1+\ e^{-\varepsilon(k)/T}}\right) dk - i\sum_{j=1}^{r} k_j^{+} + i\sum_{j=1}^{r} k_j^{-}\,,$$

$$u_i(k) = k^2-\mu+iT\sum_{j=1}^{r}\theta(k-k_j^{+}) - iT\sum_{j=1}^{r}\theta(k-k_j^{-}) - \frac{T}{2\pi}\int_{\mathbb{R}} K(k-k')\log\left(1+e^{-u_i(k')/T}\right) dk'$$

The $2r$ parameters, $\{k_j^{+}\}_{j=1}^{r}$ ($\{k_j^{-}\}_{j=1}^{r}$) are located in the upper (lower) half of the complex plane and satisfy the constraint

$$1+e^{-u_i(k_j^{\pm})/T} = 0$$

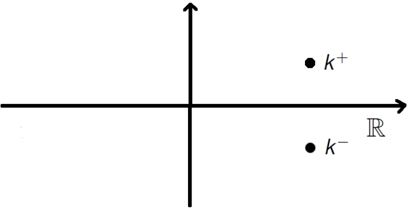
Density correlation function $\langle j(x)j(0)\rangle_T$: Leading Term

Correlation length

$$\frac{1}{\xi_0[u]} = -\frac{1}{2\pi}\int_{\mathbb{R}} \log\left(\frac{1+e^{-u(k)/T}}{1+e^{-\varepsilon(k)/T}}\right) dk - ik^+ + ik^- \,,$$

NLIE

$$u(k) = k^2 - \mu + iT\theta(k-k^+) - iT\theta(k-k^-) - \frac{T}{2\pi}\int_{\mathbb{R}} K(k-k')\log\left(1+e^{-u(k')/T}\right) dk'$$

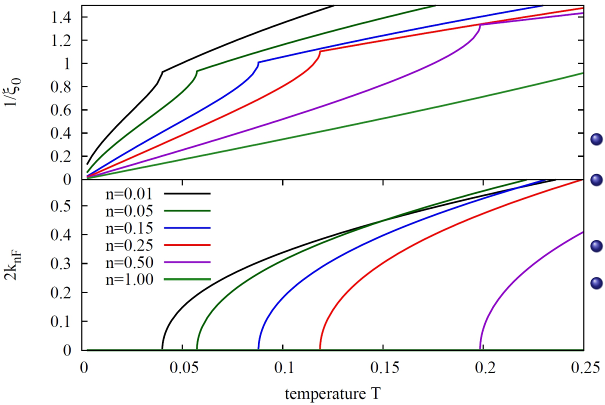


CFT expansion

$$\langle j(x)j(0)\rangle_T = \langle j(0)\rangle_T^2 - \frac{(T\mathcal{Z}/v_F)^2}{2\sinh^2(\pi T x/v_F)} + \sum_{l\in\mathbb{Z}^*} A_l\, e^{2ixlk_F}\left(\frac{\pi T/v_F}{\sinh(\pi T x/v_F)}\right)^{2l^2\mathcal{Z}^2}$$

Density correlation function $\langle j(x)j(0)\rangle_T$: Oscillatory crossover

Leading term: crossover from non-oscillating to oscillating behavior at higher T



$\gamma = c/n$

- Low-T: CFT $1/\xi_0(T) \simeq \frac{2\pi T}{v_F}$
- Above $T_o(\gamma)$ oscillations incommensurate to $2k_F$
- $\lim_{\gamma\to\infty} T_o(\gamma) = 0$
- $\lim_{\gamma\to 0} T_o(\gamma) = \infty$

At elevated temperatures distinct and real eigenvalues of the QTM merge and form complex conjugate pairs above $T_0(\gamma)$

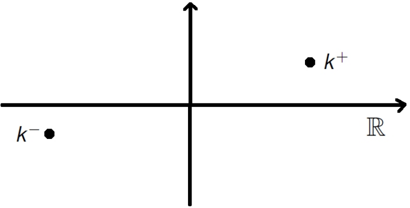
Density correlation function $\langle j(x)j(0)\rangle_T$: Next-leading term

Correlation length

$$\frac{1}{\xi_1[u]} = -\frac{1}{2\pi}\int_{\mathbb{R}} \log\left(\frac{1+e^{-u(k)/T}}{1+e^{-\varepsilon(k)/T}}\right) dk - ik^+ + ik^- ,$$

NLIE

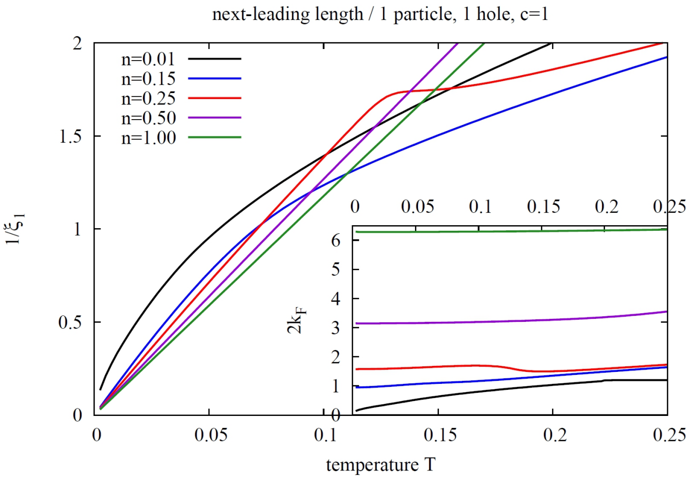
$$\overline{u}(k) = k^2 - \mu + iT\theta(k-k^+) - iT\theta(k-k^-) - \frac{T}{2\pi}\int_{\mathbb{R}} K(k-k')\log\left(1+e^{-u(k')/T}\right) dk'$$



CFT expansion

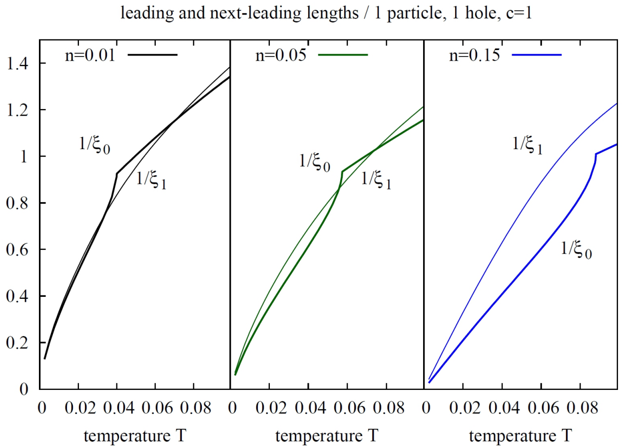
$$\langle j(x)j(0)\rangle_T = \langle j(0)\rangle_T^2 - \frac{(T\mathcal{Z}/v_F)^2}{2\sinh^2(\pi Tx/v_F)} + A_1\, e^{2ixk_F}\left(\frac{\pi T/v_F}{\sinh(\pi Tx/v_F)}\right)^{2\mathcal{Z}^2} + \cdots$$

Density correlation function $\langle j(x)j(0)\rangle_T$: Next-leading term II



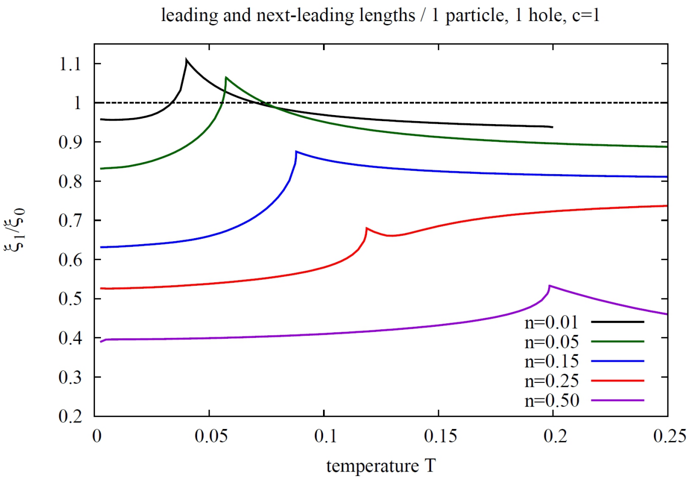
- Low-T: CFT $1/\xi_1(T) \simeq \frac{2\pi T}{v_F}\mathcal{Z}^2$ and $2k_F$ oscillations
- Region of validity for CFT decreases with γ

Density correlation function $\langle j(x)j(0)\rangle_T$: Crossover of ξ_0 and ξ_1



- $T \in [0, T_{lc}(\gamma)]$ leading correlation length ξ_0 (Im $\xi_0 = 0$)
- $T \in [T_{lc}(\gamma), T_{uc}(\gamma)]$ leading correlation length ξ_1 (Im $\xi_1 \neq 0$)
- $T \in [T_{uc}(\gamma), \infty]$ leading correlation length ξ_0 (Im $\xi_0 \neq 0$)

Density correlation function $\langle j(x)j(0)\rangle_T$: Ratio of ξ_1 and ξ_0



Similar phenomenon: The XXZ spin chain

The XXZ spin chain (no magnetic field) $\cos \pi\eta = -\Delta \in (0, 1)$ (Fabricius, Klümper and McCoy 1999)

$$H^{(0)}(\Delta) = \frac{1}{2}\sum_{j=1}^{L}\left[\sigma_x^{(j)}\sigma_x^{(j+1)} + \sigma_y^{(j)}\sigma_y^{(j+1)} + \Delta(\sigma_z^{(j)}\sigma_z^{(j+1)} - 1)\right]$$

At zero temperature

$$\langle\sigma_z^{(1)}\sigma_z^{(m+1)}\rangle_{T=0} \sim -\frac{1}{\pi^2\eta m^2} + (-1)^m\frac{C(\Delta)}{m^{1/\eta}}$$

At finite temperature

$$\langle\sigma_z^{(1)}\sigma_z^{(m+1)}\rangle_T = \sum_i A_i\left(\frac{\Lambda_i(0)}{\Lambda_0(0)}\right)^m$$

- $T < T_L(\Delta)$ leading $\Lambda_i(0)/\Lambda_0(0)$ real > 0, $A_i < 0$
- $T_L(\Delta) < T < T_U(\Delta)$ leading $\Lambda_i(0)/\Lambda_0(0)$ and A_i complex
- $T_L(\Delta) < T$ leading $\Lambda_i(0)/\Lambda_0(0)$ real > 0, $A_i > 0$

Recap

Field correlator:

- New NLIE for the correlation lengths valid for all *c* and *T*.

Density correlator:

- Oscillatory crossover present for all $\gamma \in (0, \infty)$
- No crossover for $\gamma = 0$ and $\gamma = \infty$!
- In the Tonks-Girardeau limit we find two additional crossovers in which ξ_0 and ξ_1 successively change places as the dominant correlation length

Future plans

Same method can be employed to obtain an efficient thermodynamic description of multi-component Bose and Fermi gases (Klümper and OIP, 2011)

- Amplitudes
- Correlation lengths for the 2 component Bose gas (partial results for the gas phase)
- Correlation lengths for the 2 component Fermi gas (attractive and repulsive)

THANK YOU !