Gaudin-like determinants for overlaps in integrable systems and their application to quench problems

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Integrable Lattice Models and Quantum Field Theories

Outline

- Motivation
- The spin-1/2 XXZ Heisenberg chain (fixing the notation)
- Overlap of the zero-momentum Néel state with XXZ Bethe states
  - Gaudin-like determinant formula (sketching the proof)
  - Overlaps with non-parity-invariant Bethe states
- Scaling limit to the Lieb-Liniger Bose gas
- Application to quench problems
- Conclusion and outlook

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In collaboration with... Jean-Sébastien Caux
Jacopo De Nardis
Bram Wouters
Davide Fioretto

Motivation

Why are we interested in overlaps of certain states with Bethe states?

- Combinatorial aspects of the XXZ chains (roots of unity)
- Discovering a general structure of these overlaps
- Application to non-equilibrium dynamics (quench problems)
  $\rightarrow$ Relaxation in isolated (strongly interacting) many-body quantum systems

Quench protocol:

(a) Initial state $|\Psi_0\rangle$ (not an eigenstate of the system with Hamiltonian H, e.g. ground state of a different Hamiltonian H_0; “interaction quench”)

(b) Time evolution: $|\Psi(t)\rangle = e^{-iHt}|\Psi_0\rangle$ of the state and, in particular, of observables:

$$\langle\Psi(t)|\mathcal{O}|\Psi(t)\rangle = \langle\Psi_0|e^{iHt}\mathcal{O}e^{-iHt}|\Psi_0\rangle = \sum_{m,n}\langle\Psi_0|m\rangle\langle n|\Psi_0\rangle e^{i(E_m-E_n)t}\langle m|\mathcal{O}|n\rangle$$

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$\rightarrow$ three ingredients:

1) Matrix elements $\langle m|\mathcal{O}|n\rangle$
2) Energies E_m,
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Problem: double sum over the Hilbert space: $\sum_{m,n}$

Quench protocol

(b) **Problem:** double sum over the Hilbert space: $\sum_{m,n}$

Solution:
– Restriction to a certain class of operators
(so-called "weak operators" in the thermodynamic limit)
– Applying a saddle-point approximation by minimizing the "quench action",
so-called Quench Action approach $\rightarrow$ talk by J.-S. Caux (tomorrow morning)

(c) Question about relaxation in a closed quantum many-body system can be *quantitatively* answered:
long time exp. values, relaxation process (not only for long times), exact description of the steady state $\rightarrow$ poster by J. De Nardis (tomorrow afternoon)

(d) Work was motivated by two quench scenarios:

- Interaction quench to the repulsive Lieb-Linger Bose gas starting from the ground state of the free theory ("BEC-like state"), experimentally realizable in quantum simulators, e.g. ultra cold atoms [I. Bloch et al., Rev. Mod. Phys. **80**, 885 (2008)], ...
- Quench to the spin-1/2 XXZ chain ($\Delta \geq 1$) starting from the gs of the Ising model

$$|\Psi_0\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\uparrow\downarrow\ldots\rangle + |\downarrow\uparrow\downarrow\uparrow\ldots\rangle \right)$$

[B. Wouters, MB, J. De Nardis, D. Fioretto, J.-S. Caux, arXiv:1405.0172, submitted to PRL]

Algebraic Bethe ansatz for the spin-1/2 XXZ chain

– Hamiltonian (lattice size N, $\sigma_j^\alpha =$ Pauli matrices at lattice site j):

$$H = \sum_{j=1}^{N} \Big(\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y + \Delta(\sigma_j^z \sigma_{j+1}^z - 1) \Big)$$

PBC's: $\sigma_{N+1}^\alpha = \sigma_1^\alpha$, $\alpha = x, y, z$; anisotropy parameter: $\Delta = \mathrm{ch}(\eta) = (q + q^{-1})/2$

– Yang-Baxter algebra (2×2 monodromy matrix $T(\lambda)$; λ spectral parameter):

$$\check{R}(\lambda - \mu)\,(T(\lambda) \otimes T(\mu)) = (T(\mu) \otimes T(\lambda))\,\check{R}(\lambda - \mu)$$

with R-matrix of the 6-vertex model

$$\check{R}(\lambda) = \frac{1}{\mathrm{sh}(\lambda + \eta)} \begin{pmatrix} \mathrm{sh}(\lambda+\eta) & 0 & 0 & 0 \\ 0 & \mathrm{sh}(\eta) & \mathrm{sh}(\lambda) & 0 \\ 0 & \mathrm{sh}(\lambda) & \mathrm{sh}(\eta) & 0 \\ 0 & 0 & 0 & \mathrm{sh}(\lambda+\eta) \end{pmatrix}$$

– Monodromy matrix (product in auxiliary space of N Lax operators):

$$T(\lambda) = \prod_{n=1}^{N} L_n(\lambda) = L_1(\lambda) \dots L_N(\lambda) =: \begin{pmatrix} A(\lambda) & B(\lambda) \\ C(\lambda) & D(\lambda) \end{pmatrix}.$$

Algebraic Bethe ansatz for the spin-1/2 XXZ chain

– Lax operator (2×2 matrix in auxiliary space)

$$L_n(\lambda) = \frac{1}{\mathrm{sh}(\lambda + \eta/2)} \begin{pmatrix} \mathrm{sh}\left(\lambda + \frac{\eta}{2}\sigma_n^z\right) & \mathrm{sh}(\eta)\sigma_n^- \\ \mathrm{sh}(\eta)\sigma_n^+ & \mathrm{sh}\left(\lambda - \frac{\eta}{2}\sigma_n^z\right) \end{pmatrix}$$

with Pauli matrices $\sigma_n^z, \sigma_n^\pm = \frac{1}{2}(\sigma_n^x \pm i\sigma_n^y)$ acting on lattice site n

– Transfer matrices $t(\lambda) = \mathrm{tr}_a\big(T(\lambda)\big) = A(\lambda) + D(\lambda)$ build a commutative family: $[t(\lambda), t(\mu)] = 0$

Conserved currents of the XXZ spin chain: $J_m = \frac{\partial^m}{\partial\lambda^m} \ln[t(\lambda)]\Big|_{\lambda=\eta/2}$ where $H = 2\,\mathrm{sh}(\eta) J_1$

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Conserved currents of the XXZ spin chain: $J_m = \frac{\partial^m}{\partial\lambda^m} \ln[t(\lambda)]\Big|_{\lambda=\eta/2}$ where $H = 2\operatorname{sh}(\eta)J_1$

– Pseudo vacuum $|0\rangle = |\uparrow\ldots\uparrow\rangle = |\uparrow\rangle^{\otimes N}$ $\quad\rightarrow$ monodromy matrix acts triangularly: $C(\lambda)|0\rangle = 0$

Bethe states $|\{\lambda_j\}_{j=1}^M\rangle = \prod_{j=1}^M B(\lambda_j)|0\rangle$ $\qquad$ (λ_j arbitrary = "off-shell")

Eigenstates of the transfer matrix if the parameters λ_j, $j = 1, \ldots, M$, fulfill the Bethe equations ("on-shell")

$$\left(\frac{\operatorname{sh}(\lambda_j + \eta/2)}{\operatorname{sh}(\lambda_j - \eta/2)}\right)^N = -\prod_{k=1}^M \frac{\operatorname{sh}(\lambda_j - \lambda_k + \eta)}{\operatorname{sh}(\lambda_j - \lambda_k - \eta)}, \qquad j = 1, \ldots, M.$$

Algebraic Bethe ansatz for the spin-1/2 XXZ chain

– Eigenstates of the magnetization $S^z = \sum_{n=1}^{N} \sigma_n^z/2$ with eigenvalue $N/2 - M$

Space spanned by Bethe states with fixed number M of spectral parameters: *sector of fixed magnetization* $S^z = N/2 - M$. Here: $M = N/2$

Bethe state *parity invariant* if the set of spectral parameters fulfills $\{\lambda_j\}_{j=1}^{M} = \{-\lambda_j\}_{j=1}^{M}$

– Norm of an on-shell Bethe state (Gaudin matrix G):

$$\|\{\lambda_j\}_{j=1}^{M}\| = \sqrt{\langle\{\lambda_j\}_{j=1}^{M}|\{\lambda_j\}_{j=1}^{M}\rangle},$$

$$\langle\{\lambda_j\}_{j=1}^{M}|\{\lambda_j\}_{j=1}^{M}\rangle = \operatorname{sh}^M(\eta) \prod_{\substack{j,k=1 \\ j\neq k}}^{M} \frac{\operatorname{sh}(\lambda_j - \lambda_k + \eta)}{\operatorname{sh}(\lambda_j - \lambda_k)} \det_M(G),$$

$$G_{jk} = \delta_{jk}\left(NK_{\eta/2}(\lambda_j) - \sum_{l=1}^{M} K_\eta(\lambda_j - \lambda_l)\right) + K_\eta(\lambda_j - \lambda_k),$$

where $K_\eta(\lambda) = \frac{\operatorname{sh}(2\eta)}{\operatorname{sh}(\lambda+\eta)\operatorname{sh}(\lambda-\eta)}$

[first suggested by Gaudin, McCoy, Wu (1981), then rigorously proven by Korepin (1982)]

Overlap formula – Result

- Initial state: $|\Psi_0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\uparrow\downarrow\dots\rangle + |\downarrow\uparrow\downarrow\uparrow\dots\rangle\right)$ $\leftarrow$ zero-momentum Néel state
- Overlap with XXZ <u>on-shell</u> Bethe states (main result):

$$\frac{\langle\Psi_0|\{\pm\lambda_j\}_{j=1}^{N/4}\rangle}{\|\{\pm\lambda_j\}_{j=1}^{N/4}\|} = \sqrt{2}\left[\prod_{j=1}^{N/4}\frac{\sqrt{\mathrm{th}(\lambda_j+\eta/2)\,\mathrm{th}(\lambda_j-\eta/2)}}{2\,\mathrm{sh}(2\lambda_j)}\right]\frac{\det_{N/4}(G^{(1)})}{\sqrt{\det_{N/2}(G^{(0)})}}$$

where $N/2$ even and

$$G^{(\sigma)}_{jk} = \delta_{jk}\left(NK_{\eta/2}(\lambda_j) - \sum_{l=1}^{N/4}K^{(\sigma)}_{\eta}(\lambda_j,\lambda_l)\right) + K^{(\sigma)}_{\eta}(\lambda_j,\lambda_k)\,, \quad j,k=1,\dots,N/4$$

$$K^{(\sigma)}_{\eta}(\lambda,\mu) = K_\eta(\lambda-\mu)+\sigma K_\eta(\lambda+\mu)\,, \qquad K_\eta(\lambda) = \frac{\mathrm{sh}(2\eta)}{\mathrm{sh}(\lambda+\eta)\,\mathrm{sh}(\lambda-\eta)}$$

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where $N/2$ even and

$$G_{jk}^{(\sigma)} = \delta_{jk}\left(NK_{\eta/2}(\lambda_j) - \sum_{l=1}^{N/4}K_\eta^{(\sigma)}(\lambda_j,\lambda_l)\right) + K_\eta^{(\sigma)}(\lambda_j,\lambda_k)\,, \quad j,k=1,\ldots,N/4$$

$$K_\eta^{(\sigma)}(\lambda,\mu) = K_\eta(\lambda-\mu) + \sigma K_\eta(\lambda+\mu)\,, \qquad K_\eta(\lambda) = \frac{\mathrm{sh}(2\eta)}{\mathrm{sh}(\lambda+\eta)\,\mathrm{sh}(\lambda-\eta)}$$

Remarks:

- Bethe states are parity invariant: $\{\lambda_j\}_{j=1}^{N/2} = \{-\lambda_j\}_{j=1}^{N/2} \equiv \{\pm\lambda_j\}_{j=1}^{N/4}$
- Bethe roots complex numbers (string solutions)
- The case $N/2$ odd can be treated similarly.

  [MB, J. De Nardis, B. Wouters, J.-S. Caux, arXiv:1403.7469]
- Overlaps with non-parity-invariant Bethe states vanish.

Overlap formula – Sketch of the proof (Part I)

First step: Getting a determinant formula [Tsuchiya (1998), Pozsgay, Kozlowski (2012)]

Main ideas:

- Consider a 6-vertex model with reflecting ends (reflection equation needed)
- Define partition function that (after a simple transformation) turns into the overlap of a Bethe state with a certain boundary state (= product state of local two-site states)
- Calculate the partition function

Result ($\widetilde{\lambda}_j$ arbitrary(!) complex numbers, $s_{x,y} = \mathrm{sh}(x+y)$, $M = N/2$)**:**

$$\langle \Psi_0 | \{\widetilde{\lambda}_j\}_{j=1}^M \rangle = \sqrt{2} \left[\prod_{j=1}^{M} \frac{s_{\widetilde{\lambda}_j,+\eta/2}}{s_{2\widetilde{\lambda}_j,0}} \frac{s^M_{\widetilde{\lambda}_j,-\eta/2}}{s^M_{\widetilde{\lambda}_j,+\eta/2}} \right] \left[\prod_{j>k=1}^{M} \frac{s_{\widetilde{\lambda}_j+\widetilde{\lambda}_k,\eta}}{s_{\widetilde{\lambda}_j+\widetilde{\lambda}_k,0}} \right] \det_M(1+U)$$

$$U_{jk} = \frac{s_{2\widetilde{\lambda}_k,\eta}\, s_{2\widetilde{\lambda}_k,0}}{s_{\widetilde{\lambda}_j+\widetilde{\lambda}_k,0}\, s_{\widetilde{\lambda}_j-\widetilde{\lambda}_k,\eta}} \left[\prod_{\substack{l=1 \\ l\neq k}}^{M} \frac{s_{\widetilde{\lambda}_k+\widetilde{\lambda}_l,0}}{s_{\widetilde{\lambda}_k-\widetilde{\lambda}_l,0}} \right] \left[\prod_{l=1}^{M} \frac{s_{\widetilde{\lambda}_k-\widetilde{\lambda}_l,-\eta}}{s_{\widetilde{\lambda}_k+\widetilde{\lambda}_l,+\eta}} \right] \left(\frac{s_{\widetilde{\lambda}_k,+\eta/2}}{s_{\widetilde{\lambda}_k,-\eta/2}} \right)^N$$

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$$U_{jk} = \frac{s_{2\widetilde{\lambda}_k, \eta}\, s_{2\widetilde{\lambda}_k, 0}}{s_{\widetilde{\lambda}_j + \widetilde{\lambda}_k, 0}\, s_{\widetilde{\lambda}_j - \widetilde{\lambda}_k, \eta}} \left[\prod_{\substack{l=1 \\ l \neq k}}^M \frac{s_{\widetilde{\lambda}_k + \widetilde{\lambda}_l, 0}}{s_{\widetilde{\lambda}_k - \widetilde{\lambda}_l, 0}} \right] \left[\prod_{l=1}^M \frac{s_{\widetilde{\lambda}_k - \widetilde{\lambda}_l, -\eta}}{s_{\widetilde{\lambda}_k + \widetilde{\lambda}_l, +\eta}} \right] \left(\frac{s_{\widetilde{\lambda}_k, +\eta/2}}{s_{\widetilde{\lambda}_k, -\eta/2}} \right)^N$$

Remarks:

- expression inconvenient to perform the thermodynamic limit
- singularities in the prefactor + zeroes of the determinant for parity-invariant states
- But: expression valid for *off-shell* Bethe states
- Idea: perform the limit to parity-invariant states (not necessarily on-shell Bethe states)

Overlap formula – Sketch of the proof (Part II)

Reducing the determinant (off-shell formula):

- Perform the limit to parity-invariant off-shell states
- Set $\widetilde{\lambda}_j = \lambda_j + \varepsilon_j$ for $j = 1, \ldots, M/2$ and $\widetilde{\lambda}_j = -\lambda_{j-M/2} + \varepsilon_{j-M/2}$ for $j = M/2+1, \ldots, M$ with arbitrary complex numbers λ_j, $j = 1, \ldots, M/2$
- Main ingredients of the proof:
  - $\varepsilon_j \to 0$, $j = 1, \ldots, M/2$
  - pseudo parity invariance of the set $\{\widetilde{\lambda}_j\}_{j=1}^{M} = \{\lambda_j + \varepsilon_j\}_{j=1}^{M/2} \cup \{-\lambda_j + \varepsilon_j\}_{j=1}^{M/2}$
- Bethe equations are less important (only at the very end)

Simple determinant manipulations and expanding everything carefully in small ε_j:

$\det_M[1+U] =$

$$\det_M \begin{pmatrix} \begin{bmatrix} \varepsilon_1 D_1 & 0 \\ 0 & 1 \end{bmatrix} & \begin{bmatrix} \varepsilon_2 e_{12} & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} \varepsilon_3 e_{13} & 0 \\ 0 & 0 \end{bmatrix} & \cdots \\ \begin{bmatrix} \varepsilon_1 e_{21} & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} \varepsilon_2 D_2 & 0 \\ 0 & 1 \end{bmatrix} & \begin{bmatrix} \varepsilon_3 e_{23} & 0 \\ 0 & 0 \end{bmatrix} & \\ \begin{bmatrix} \varepsilon_1 e_{31} & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} \varepsilon_2 e_{32} & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} \varepsilon_3 D_3 & 0 \\ 0 & 1 \end{bmatrix} & \\ \vdots & & & \ddots \end{pmatrix} = \left[\prod_{j=k}^{M/2} \varepsilon_k\right] \det_{M/2} \begin{bmatrix} D_1 & e_{12} & e_{13} & \ldots \\ e_{21} & D_2 & e_{23} & \\ e_{31} & e_{32} & D_3 & \\ \vdots & & & \ddots \end{bmatrix}$$

Overlap formula – Sketch of the proof (Part II)

where ($\alpha_k = \sqrt{-\frac{s_{2\lambda_k,+\eta}}{s_{2\lambda_k,-\eta}}\mathfrak{a}_k}$, and $\mathfrak{b}_k^\pm$ first order corrections of $U_{2k-1,2k}$, $U_{2k,2k-1}$, respectively):

$$D_k = \lim_{\{\varepsilon_k\to 0\}_{k=1}^{M/2}}(\ldots) = -\frac{s_{2\lambda_k,-\eta}}{s_{2\lambda_k,0}}\alpha_k^2 - \frac{s_{2\lambda_k,+\eta}}{s_{2\lambda_k,0}}\alpha_k^{-2} - \mathfrak{b}_k^+\alpha_k^{-2} - \mathfrak{b}_k^-\alpha_k^2$$

$$= \frac{s_{2\lambda_k,+\eta}}{s_{2\lambda_k,0}}\mathfrak{a}_k + \frac{s_{2\lambda_k,-\eta}}{s_{2\lambda_k,0}}\mathfrak{a}_k^{-1} + 2\,\mathrm{ch}(\eta) - s_{0,\eta}\partial_{\lambda_k}\ln\left\{\frac{s^{2M}_{\lambda_k,+\eta/2}}{s^{2M}_{\lambda_k,-\eta/2}}\prod_{\substack{l=1\\l\neq k}}^{M/2}\prod_{\sigma=\pm}\frac{s_{\lambda_k+\sigma\lambda_l,-\eta}}{s_{\lambda_k+\sigma\lambda_l,+\eta}}\right\}$$

$$= \frac{s_{2\lambda_k,+\eta}}{s_{2\lambda_k,0}}\mathfrak{A}_k + \frac{s_{2\lambda_k,-\eta}}{s_{2\lambda_k,0}}\bar{\mathfrak{A}}_k + 2Ms_{0,\eta}K_{\eta/2}(\lambda_k) - \sum_{\substack{l=1\\l\neq k}}^{M/2} s_{0,\eta}K_\eta^+(\lambda_k,\lambda_l)$$

$$e_{jk} = \sqrt{\frac{s_{2\lambda_k,+\eta}s_{2\lambda_k,-\eta}\mathfrak{a}_j}{s_{2\lambda_j,+\eta}s_{2\lambda_j,-\eta}\mathfrak{a}_k}}\left(K_\eta^+(\lambda_j,\lambda_k) + f_{jk}\right)$$

where $K_\eta^+(\lambda,\mu) = K_\eta(\lambda-\mu) + K_\eta(\lambda+\mu)$, $K_\eta(\lambda) = \frac{s_{0,2\eta}}{s_{\lambda,+\eta}s_{\lambda,-\eta}}$ and $\mathfrak{A}_k = 1+\mathfrak{a}_k$, $\bar{\mathfrak{A}}_k = 1+\mathfrak{a}_k^{-1}$
where

$$\mathfrak{a}_k = \mathfrak{a}(\lambda_k) = \left[\prod_{\substack{l=1\\\sigma=\pm}}^{M/2}\frac{s_{\lambda_k-\sigma\lambda_l,-\eta}}{s_{\lambda_k-\sigma\lambda_l,+\eta}}\right]\left(\frac{s_{\lambda_k,+\eta/2}}{s_{\lambda_k,-\eta/2}}\right)^{2M}$$

Overlap formula – Sketch of the proof (Part II)

where ($\alpha_k = \sqrt{-\frac{s_{2\lambda_k,+\eta}}{s_{2\lambda_k,-\eta}}\mathfrak{a}_k}$, and $\mathfrak{b}_k^{\pm}$ first order corrections of $U_{2k-1,2k}$, $U_{2k,2k-1}$, respectively):

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$$= \frac{s_{2\lambda_k,+\eta}}{s_{2\lambda_k,0}}\mathfrak{a}_k + \frac{s_{2\lambda_k,-\eta}}{s_{2\lambda_k,0}}\mathfrak{a}_k^{-1} + 2\,\mathrm{ch}(\eta) - s_{0,\eta}\partial_{\lambda_k}\ln\left\{\frac{s^{2M}_{\lambda_k,+\eta/2}}{s^{2M}_{\lambda_k,-\eta/2}}\prod_{\substack{l=1\\l\neq k}}^{M/2}\prod_{\sigma=\pm}\frac{s_{\lambda_k+\sigma\lambda_l,-\eta}}{s_{\lambda_k+\sigma\lambda_l,+\eta}}\right\}$$

$$= \frac{s_{2\lambda_k,+\eta}}{s_{2\lambda_k,0}}\mathfrak{A}_k + \frac{s_{2\lambda_k,-\eta}}{s_{2\lambda_k,0}}\bar{\mathfrak{A}}_k + 2Ms_{0,\eta}K_{\eta/2}(\lambda_k) - \sum_{\substack{l=1\\l\neq k}}^{M/2} s_{0,\eta}K_\eta^{+}(\lambda_k,\lambda_l)$$

$$e_{jk} = \sqrt{\frac{s_{2\lambda_k,+\eta}s_{2\lambda_k,-\eta}\mathfrak{a}_j}{s_{2\lambda_j,+\eta}s_{2\lambda_j,-\eta}\mathfrak{a}_k}}\left(K_\eta^{+}(\lambda_j,\lambda_k) + f_{jk}\right)$$

where $K_\eta^{+}(\lambda,\mu) = K_\eta(\lambda-\mu) + K_\eta(\lambda+\mu)$, $K_\eta(\lambda) = \frac{s_{0,2\eta}}{s_{\lambda,+\eta}s_{\lambda,-\eta}}$ and $\mathfrak{A}_k = 1+\mathfrak{a}_k$, $\bar{\mathfrak{A}}_k = 1+\mathfrak{a}_k^{-1}$
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After further manipulations... finally...

Overlap formula – Sketch of the proof (Part II)

Off-shell overlap formula (λ_j arbitrary complex numbers):

$$\langle\Psi_0|\{\pm\lambda_j\}_{j=1}^{M/2}\rangle = \langle\Psi_0|\{\lambda_j+\varepsilon_j\}_{j=1}^{M/2}\cup\{-\lambda_j+\varepsilon_j\}_{j=1}^{M/2}\rangle\Big|_{\{\varepsilon_j\to 0\}_{j=1}^{M/2}} = \gamma \det{}_{M/2}(G^+)\,,$$

where

$$\gamma = \sqrt{2}\left[\prod_{j=1}^{M/2}\frac{s_{\lambda_j,+\eta/2}^{2M+1}s_{\lambda_j,-\eta/2}^{2M+1}}{s_{2\lambda_j,0}^2}\right]\left[\prod_{\substack{j>k=1\\ \sigma=\pm}}^{M/2}\frac{s_{\lambda_j+\sigma\lambda_k,+\eta}s_{\lambda_j+\sigma\lambda_k,-\eta}}{s_{\lambda_j+\sigma\lambda_k,0}^2}\right]$$

$$G_{jk}^+ = \delta_{jk}\left(N s_{0,\eta}K_{\eta/2}(\lambda_j) - \sum_{l=1}^{M/2} s_{0,\eta}K_\eta^+(\lambda_j,\lambda_l)\right) + s_{0,\eta}K_\eta^+(\lambda_j,\lambda_k)$$
$$+\delta_{jk}\frac{s_{2\lambda_j,+\eta}\mathfrak{A}_j + s_{2\lambda_j,-\eta}\bar{\mathfrak{A}}_j}{s_{2\lambda_j,0}} + (1-\delta_{jk})f_{jk}\,, \qquad j,k=1,\ldots,M/2$$

$$f_{jk} = \mathfrak{A}_k\left(\frac{s_{2\lambda_j,+\eta}s_{0,\eta}}{s_{\lambda_j+\lambda_k,0}s_{\lambda_j-\lambda_k,+\eta}} - \frac{s_{2\lambda_j,-\eta}s_{0,\eta}}{s_{\lambda_j-\lambda_k,0}s_{\lambda_j+\lambda_k,-\eta}}\right) + \mathfrak{A}_k\bar{\mathfrak{A}}_j\frac{s_{2\lambda_j,-\eta}s_{0,\eta}}{s_{\lambda_j-\lambda_k,0}s_{\lambda_j+\lambda_k,-\eta}}$$
$$-\bar{\mathfrak{A}}_j\left(\frac{s_{2\lambda_j,-\eta}s_{0,\eta}}{s_{\lambda_j-\lambda_k,0}s_{\lambda_j+\lambda_k,-\eta}} + \frac{s_{2\lambda_j,-\eta}s_{0,\eta}}{s_{\lambda_j+\lambda_k,0}s_{\lambda_j-\lambda_k,-\eta}}\right)$$

Overlap formula – Result

- Initial state: $|\Psi_0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\uparrow\downarrow\ldots\rangle + |\uparrow\downarrow\uparrow\downarrow\ldots\rangle\right)$ $\leftarrow$ zero-momentum Néel state
- Overlap with XXZ *on-shell* Bethe states (main result):

$$\frac{\langle\Psi_0|\{\pm\lambda_j\}_{j=1}^{N/4}\rangle}{\|\{\pm\lambda_j\}_{j=1}^{N/4}\|} = \sqrt{2}\left[\prod_{j=1}^{N/4}\frac{\sqrt{\mathrm{th}(\lambda_j+\eta/2)\,\mathrm{th}(\lambda_j-\eta/2)}}{2\,\mathrm{sh}(2\lambda_j)}\right]\frac{\det_{N/4}(G^{(1)})}{\sqrt{\det_{N/2}(G^{(0)})}}$$

where

$$G_{jk}^{(\sigma)} = \delta_{jk}\left(NK_{\eta/2}(\lambda_j) - \sum_{l=1}^{N/4}K_\eta^{(\sigma)}(\lambda_j,\lambda_l)\right) + K_\eta^{(\sigma)}(\lambda_j,\lambda_k)\,, \quad j,k = 1,\ldots,N/4$$

$$K_\eta^{(\sigma)}(\lambda,\mu) = K_\eta(\lambda-\mu) + \sigma K_\eta(\lambda+\mu)\,, \qquad K_\eta(\lambda) = \frac{\mathrm{sh}(2\eta)}{\mathrm{sh}(\lambda+\eta)\,\mathrm{sh}(\lambda-\eta)}$$

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Remarks:

- Bethe roots complex numbers (string solutions)
- Bethe states are parity invariant: $\{\lambda_j\}_{j=1}^{N/2} = \{-\lambda_j\}_{j=1}^{N/2} \equiv \{\pm\lambda_j\}_{j=1}^{N/4}$
- Overlaps with non-parity-invariant Bethe states vanish.
  [MB, J. De Nardis, B. Wouters, J.-S. Caux, arXiv:1403.7469]
- The case $N/2$ odd can be treated similarly.

Scaling limit to the LL Bose gas

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- Scaling limit:

$$\eta = \mathrm{i}\pi - \mathrm{i}\varepsilon\,, \qquad N = cL/\varepsilon^2\,, \qquad \lambda_j \to \varepsilon\lambda_j/c\,, \qquad \varepsilon \to 0 \qquad \left(\Delta = \mathrm{ch}(\eta) = \frac{q+q^{-1}}{2} \to -1\right)$$

- Bethe equations (for a finite(!) number $M = N_{LL}$ of rapidities, N_{LL} even):

$$e^{iL\lambda_j} = -\prod_{k=1}^{N_{LL}} \frac{\lambda_j - \lambda_k + ic}{\lambda_j - \lambda_k - ic}\,, \qquad j = 1, \ldots, N_{LL}$$

These are the Bethe equations of the Lieb-Linger Bose gas [Lieb and Liniger 1963]:

$$H_{LL} = -\sum_{j=1}^{N_{LL}} \frac{\partial^2}{\partial x_j^2} + 2c \sum_{j>k} \delta(x_j - x_k)$$

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Problem:

- Here: $M = N/2$ flipped spins in the Néel state $\to$ M not finite
- Matrix in the determinant of the overlap formula is $N \times N$, becomes infinite dimensional

Solution:

- Flip (infinitely many) spins, respecting the symmetry of the model AND parity invariance
- Use $B(\lambda)$ and $C(\lambda)$ operators for large spectral parameter ($\lambda \to \pm\infty$)

Flipping spins – $U_q(\hat{sl}_2)$ raising operators

- Calculate B- and C-operators in the limit $\lambda \to \pm\infty$
- Monodromy matrix for large $\lambda \to \pm\infty$ ($q = e^{\eta}$, $s_n^z = \sigma_n^z/2$, $s_n^{\pm} = \sigma_n^{\pm}$):

$$\begin{pmatrix} A(\lambda) & B(\lambda) \\ C(\lambda) & D(\lambda) \end{pmatrix} \sim q^{\mp N/2} \prod_{n=1}^{N} \left[\begin{pmatrix} q^{\pm s_n^z} & 0 \\ 0 & q^{\mp s_n^z} \end{pmatrix} \pm 2 e^{\mp\lambda} \mathrm{sh}(\eta) \begin{pmatrix} 0 & s_n^- \\ s_n^+ & 0 \end{pmatrix} \right]$$

- Spin raising and lowering operators ($U_q(\hat{sl}_2)$ symmetry) [Pasquier (1990)]:

$$S_q^{\mp} = \lim_{\lambda \to \pm\infty} \left(\frac{q^{\pm N/2} \mathrm{sh}(\lambda) \{B/C\}(\lambda)}{\mathrm{sh}(\eta)} \right) = \sum_{n=1}^{N} \left[\prod_{j=1}^{n-1} q^{+s_j^z} \right] s_n^{\mp} \left[\prod_{j=n+1}^{N} q^{-s_j^z} \right]$$

$$\widetilde{S}_q^{\mp} = \lim_{\lambda \to \mp\infty} \left(\frac{q^{\mp N/2} \mathrm{sh}(\lambda) \{B/C\}(\lambda)}{\mathrm{sh}(\eta)} \right) = \sum_{n=1}^{N} \left[\prod_{j=1}^{n-1} q^{-s_j^z} \right] s_n^{\mp} \left[\prod_{j=n+1}^{N} q^{+s_j^z} \right]$$

- q-raised Néel states ($n = N/4 - N_{LL}/2$):

$$|\Psi_0^{(n)}\rangle = \left(S_q^+ \widetilde{S}_q^+ \right)^n |\Psi_0\rangle \qquad \text{and} \qquad \langle\Psi_0^{(n)}| = \langle\Psi_0| \left(S_q^- \widetilde{S}_q^- \right)^n$$

- $|\uparrow\downarrow\uparrow\downarrow\ldots\rangle \quad \to \quad \sum |\uparrow\uparrow\ldots\uparrow\downarrow\uparrow\uparrow\ldots\uparrow\uparrow\downarrow\uparrow\ldots\uparrow\rangle$ (N_{LL} spins pointing down)
- Limits $q \to -1$, $N \to \infty$: no problems with periodic boundary conditions

Overlap formula in the scaling limit – Overlap of BEC with LL Bethe states

- 1. Start with the overlap of the Néel state with an XXZ **off-shell** Bethe state
  2. Send n many rapidities to $+\infty$, n many to $-\infty$ (parity invariance of the state!)
  3. Perform the scaling limit (using that $S_q^\pm$, $\widetilde{S}_q^\pm$ become $SU(2)$ symmetry operators)

  Then (after a straightforward calculation [MB, J. Stat. Mech. (2014) P05006]):

$$\frac{\langle BEC|\{\pm\lambda_j\}_{j=1}^{N_{LL}/2}\rangle}{\|\{\pm\lambda_j\}_{j=1}^{N_{LL}/2}\|} = \frac{\sqrt{(cL)^{-N_{LL}}N_{LL}!}}{\prod_{j=1}^{N_{LL}/2}\frac{\lambda_j}{c}\sqrt{\frac{\lambda_j^2}{c^2}+\frac{1}{4}}}\,\frac{\det_{N_{LL}/2}(\widetilde{G}^{(1)})}{\sqrt{\det_{N_{LL}}(\widetilde{G}^{(0)})}}$$

$$\widetilde{G}_{jk}^{(\sigma)} = \delta_{jk}\Big(L+\sum_{l=1}^{N_{LL}/2}\widetilde{K}^{(|\sigma|)}(\lambda_j,\lambda_l)\Big) - \widetilde{K}^{(\sigma)}(\lambda_j,\lambda_k)\,,$$

$$\widetilde{K}^{(\sigma)}(\lambda,\mu) = \widetilde{K}(\lambda-\mu)+\sigma\widetilde{K}(\lambda+\mu)\,,\quad \widetilde{K}(\lambda) = 2c/(\lambda^2+c^2)$$

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- **Four Remarks:**
  1. Normalized LL overlap formula is only valid “on-shell”,
     because of the norm determinant in the denominator: $\det_{N_{LL}}(\widetilde{G}^{(0)})$.
  2. There is a “reduced” determinant expression for overlaps of the q-raised Néel state with unnormalized *off-shell* Bethe states (before and after taking the scaling limit).
  3. To get the correct pre-factor one has to count all states that scale to the same state.
  4. The sum over all spin distributions give the BEC state: $\langle \mathbf{x}|BEC\rangle = L^{-N_{LL}/2}$

Application to quench problems

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Procedure:

- Quench action approach (see talk by J-S)
- Leading part of the overlap in the thermodynamic limit
- Ratio of determinants $\left(\frac{\det_{N/2}(G^{(1)})}{\sqrt{\det_N(G^{(0)})}} = \sqrt{\frac{\det_{N/2}(G^{(+1)})}{\det_{N/2}(G^{(-1)})}}\right)$ is subleading. (Always?)
- Prefactor directly translates into the "driving term(s)" of the generalized TBA equation(s)

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Example: Lieb-Liniger [J. De Nardis, B. Wouters, MB, J.-S. Caux, PRA 89 (2014) 033601]
(Spin-1/2 XXZ massive in [B. Wouters, MB, J. De Nardis, D. Fioretto, J.-S. Caux, arXriv:1405.0172])

- Prefactor (in the thermodynamic limit $L \to \infty$, $N_{LL} \to \infty$, $n = N_{LL}/L$ fixed, $x_j := \lambda_j/c$):

$$2 \log \left[\sqrt{(cL)^{-N_{LL}} N_{LL}!} \Big/ \prod_{j=1}^{N_{LL}/2} \frac{\lambda_j}{c} \sqrt{\frac{\lambda_j^2}{c^2} + \frac{1}{4}} \right] \to -\log\left[x^2(x^2+1/4)\right]$$

- gTBA equation: $\log a(x) = \log(\tau^2) - \log\left[x^2(x^2+1/4)\right] + \int_{-\infty}^{\infty} K(x-y) \log[1+a(y)]\, dy$
- Explicit solution: $a(x) = \frac{2\pi\tau}{x\,\mathrm{sh}(2\pi x)} I_{1-2ix}(4\sqrt{\tau}) I_{1+2ix}(4\sqrt{\tau})$, and $\tau = 1/\gamma = c/n$

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Remark: The gTBA equations for XXZ also have an explicit solution!

Summary and outlook

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- Application to interaction quench problems:
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    for correlation functions see [J. De Nardis, B. Wouters, MB, J.-S. Caux, PRA 89 (2014) 033601]
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Thank you for your attention!